Quantum General Relativity and Effective

量子广义相对论与有效

Field Theory

场论

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摘要

This is a review of some of the concepts and results of the effective field theory treatment of quantum general relativity. Included are lessons of low-energy quantum gravity and a discussion of the limits of effective field theory techniques.

本文综述了量子广义相对论的有效场论处理的部分概念与研究结果，包含低能量子引力的经验总结，同时讨论了有效场论方法的局限性。

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Introduction

引言

Perhaps without realizing it, we have lived through a paradigm change in the way that we understand the fundamental interactions. Historically, we started out uncovering what we call classical physics and then found awkward ways to describe quantum physics. The quantum techniques themselves have evolved and are now different than even 30 years ago. The logic of effective field theory now permeates the field. We also now give primacy to the quantum theory and have some modest understanding of how the classical world emerges from the quantum world. This shift in our understanding is particularly important in the case of general relativity. The earliest quantum techniques did not work well for general relativity. However, the modern viewpoint is well suited for gravity. We have a theory of quantum general relativity which treats it, within various limits, as an effective field theory. This brief review is a chance to reflect on the basic ideas and lessons of this effective field theory.

或许我们都未曾察觉，我们已然亲历了人类理解基本相互作用方式的范式转变。从历史发展来看，我们先是逐步揭示了经典物理学，而后才摸索出曲折的方法来描述量子物理学。如今量子技术本身已经历发展，面貌与三十年前截然不同。有效场论的逻辑现已渗透整个领域。我们如今也将量子理论置于首要地位，并且对经典世界如何从量子世界中涌现有了初步的认识。这种认知转变在广义相对论领域尤为重要。早期的量子方法在处理广义相对论时效果不佳，但现代视角非常适配引力研究。我们现有的量子广义相对论理论在多种极限下将引力作为有效场论处理。这篇简要综述正是一次梳理有效场论核心思想与核心结论的机会。

Our fundamental theory is now defined by a path integral over the dynamical degrees of freedom guided by a local Lagrangian. Physics is an experimental science, and through much effort, we have determined the particles and the structure of their interactions. In a compact notation (and hiding about 26 parameters - masses, couplings, etc.), our core theory is presently given by

我们的基础理论如今由动力学自由度的路径积分定义，其核心是局域拉格朗日量。物理学是实验科学，通过大量研究，我们已经确定了基本粒子及其相互作用的结构。用紧凑记号表示 (还隐去了约 26 个参数，即质量、耦合常数等)，我们目前的核心理论可写为

$$Z^{\text{core}} = \int [d\phi d\psi dA dg]_\Lambda e^{i\int d^4x\left[-\frac{1}{4}F^2+\overline{\psi}i\not{D}\psi+\frac{1}{2}\partial\phi\partial\phi-V(\phi)-\Gamma\overline{\psi}\phi\psi-\Lambda_{cc}+\frac{2}{\kappa^2}R+\ldots\right]}.$$

(1)

We use the path integral because canonical quantization is not a useful way to treat the gauge interactions of the Standard Model, while path integrals are simple and direct. Here, the subscript Λ is meant to indicate that the path integral is to be restricted to those energies which we have experimentally explored - i.e., those below some scale Λ . We do not, and need not, pretend that these degrees of freedom are always the correct ones. The ellipses indicate that we expect further local terms in the action, suppressed by heavy mass scales, even if we have not identified them yet. That is, we think of all of our core theory as an effective field theory valid at ordinary energies.

我们使用路径积分是因为正则量子化不适合处理标准模型的规范相互作用，而路径积分简洁直接。此处，下标 Λ 表示路径积分仅限定于我们已实验探测到的能量范围，即低于某个能标 Λ 。我们不会也无需假定这些自由度永远都是正确的。式中的省略号表示，即使尚未确定这些项，我们也预期作用量中还存在更多被大质量标度压低的局域项。也就是说，我们将整个核心理论视为在常规能区成立的有效场论。

From this starting point, general relativity is also fundamentally a quantum theory. The metric degrees in freedom need to be dynamical, and they need to be included in the path integral because otherwise we could not obtain the classical physics such as gravitational waves.

从这一起点出发，广义相对论本质上也是量子理论。度规自由度必须是动力学的，并且需要被纳入路径积分，否则我们无法得到引力波这类经典物理结果。

The idea of effective field theory is that the low-energy degrees of freedom organize themselves as quantum fields, governed by a local Lagrangian, in general containing so-called nonrenormalizable terms suppressed by powers of a heavy scale. Nevertheless, one can make predictions without knowledge of the full high-energy theory.

有效场论的核心思想是: 低能自由度会以量子场的形式存在，遵从局域拉格朗日量，拉格朗日量中一般包含所谓的不可重整化项，这些项会被重标度的幂次压低。即便如此，我们无需知晓完整的高能理论也可以做出预言。

The basic theme of this exposition is that general relativity is a quantum field theory which is not much different than the other interactions in our core theory. It is explored using the tools of effective field theory. In the region of validity of the effective field theory, it can be studied in perturbation theory, and the quantum corrections are small - it is the best perturbative theory ever. However, the lessons learned are nevertheless interesting.

本文的核心主旨是: 广义相对论是一种量子场论，它和我们核心理论中的其他相互作用并没有太大差异，我们可以用有效场论工具对其进行研究。在有效场论的适用范围内，我们可以用微扰论进行研究，且量子修正很小——它是有史以来最优秀的微扰理论。即便如此，我们从中得到的结论仍然十分有意义。

In other reviews, I have presented more of the technical details of the general relativistic effective field theory (GREFT) [1-5] (see also [6,7]), as well as other expositions on effective field theory [8-11]. Here, I attempt a broader overview.

在我过往的其他综述中，已经介绍了广义相对论有效场论 (GREFT) 更多的技术细节 [1-5](也可参见 [6,7]，以及我关于有效场论的其他阐述 [8-11])。本文中，我将尝试给出一个更宽泛的概述。

Does the Graviton Exist?

引力子存在吗?

The reader who finds this question annoying or boring is free to skip this section. However, I include it because it bears on the rationale for effective field theory.

觉得这个问题无聊烦人、不想看的读者大可跳过本节。但我仍保留本节，因为它关系到有效场论的基本原理。

One often hears the question of whether the graviton exists. Some people will never be satisfied until they see the clicks in a detector caused by a graviton. With this criterium, there will not be a resolution in our lifetimes or perhaps ever. But for some the question means to ask whether it would be consistent to have everything but gravity be described by quantum fields, but to have gravity be classical. As we presently understand quantum theory, this is not possible. The following brief comments illustrate how the existence of quantum fields is required.

我们经常听到引力子是否存在的疑问。有些人非要亲眼看到探测器里引力子产生的计数才会满意。按照这个标准，在我们有生之年，甚至可能永远都得不到答案。但对另一些人来说，这个问题其实是在问: 除引力外所有相互作用都用量子场描述，唯独引力是经典的，这套理论是否自洽？就我们目前对量子理论的理解，这不可能成立。以下简要说明为什么引力也必须是量子场。

Steven Weinberg in particular has presented the argument that any quantum theory satisfying Lorentz invariance, causality, crossing symmetry, and cluster decomposition will be described by a quantum field theory [8, 9]. This is part of the reasoning that all of our theories are effective field theories. Let us see how this could be applied to the gravitational interaction (This form of the argument comes from D. Carney [12].). The Newtonian potential has the Fourier transform

特别是史蒂文·温伯格，他提出了一个论证: 任何满足洛伦兹不变性、因果性、交叉对称性和集团分解原理的量子理论，都必然是量子场论 [8,9]。这是我们认为所有现有理论都是有效场论的核心依据之一。我们来看这个论证如何应用到引力相互作用上 (该论证形式来自 D. 卡尼 [12])。牛顿势的傅里叶变换为

$$-\frac{1}{\mathbf{q}^2} = -\frac{1}{(\mathbf{p}_1 - \mathbf{p}_2)^2}. \tag{2}$$

But for this to be consistent with special relativity, this must be made into a Lorentz invariant.

但要让这个结果满足狭义相对论，必须将其改写成洛伦兹不变的形式。

$$-\frac{1}{(\mathbf{p}_1 - \mathbf{p}_2)^2} \to \frac{1}{(p_1 - p_2)_\mu (p_1 - p_2)^\mu} = \frac{1}{(p_1 - p_2)^2} = \frac{1}{q^2}. \tag{3}$$

We now need to specify how we deal with the pole in this function. Of course, we know that the correct answer is the Feynman propagator

现在我们需要明确如何处理这个函数的极点。当然我们都知道正确答案是费曼传播子

$$\frac{1}{q^2 + i\varepsilon} \tag{4}$$

and it is this which emerges from the path integral treatments. However, if we are trying to be more general, Carney [12] has shown that alternate prescriptions with retarded or advanced Green functions, i.e., with $(q_0 \pm i\varepsilon)^2 - \mathbf{q}^2$, do not satisfy unitarity, nor do these forms satisfy causality. The propagator using $-i\varepsilon$ is equivalent to the one with $+i\varepsilon$, although the arrow of causality is reversed [13,14]. But now we are basically done. Because of the identity

这一点正是从路径积分处理中得出的。然而，如果我们尝试进行更广义的分析，卡尼 [12] 已经证明，采用推迟格林函数或超前格林函数的替代方案 (即含 $(q_0 \pm i\varepsilon)^2 - \mathbf{q}^2$ 的方案) 不满足幺正性，这些形式也不满足因果性。使用 $-i\varepsilon$ 的传播子等价于使用 $+i\varepsilon$ 的传播子，只是因果箭头是反向的 [13,14]。但到这里我们的推导已经基本完成。根据恒等式

$$\frac{1}{q^2 + i\varepsilon} = P\frac{1}{q^2} - i\pi\delta\left(q^2\right) \tag{5}$$

we see that what originally was the potential comes accompanied by massless on-shell radiation - the graviton. This combination yields graviton exchange and graviton emission. The general principles of relativity, unitarity, and causality have turned the potential into a quantum propagator.

我们可以看到，原本的势必然伴随着无质量的在壳辐射——也就是引力子。这个组合给出了引力子交换和引力子辐射过程。相对论、幺正性和因果性这些基本原理，已经把牛顿势转化为量子传播子。

This conclusion can also be addressed in different ways. As a recent example, the authors of Ref. [15, 16] show that in order to resolve a gedanken experiment involving superposed charges or masses, in electromagnetism or gravity, requires the existence of radiation - photons or gravitons. There are several previous arguments which say that for consistency, the gravitational potential must also be accompanied by gravitons [17-19].

这个结论也可以通过其他方式推导。比如最近的工作，文献 [15,16] 的作者指出，无论是电磁还是引力情形，要解答涉及叠加电荷或叠加质量的思想实验，都要求存在辐射——也就是光子或引力子。此前也已有多个论证指出，为了自洽，引力势必须伴随引力子存在 [17-19]。

We have also partially addressed this issue using the path integral mentioned in the introduction. The metric appears in the Lagrangian because we need to describe particles in curved space. With the rest of the interactions described by the path integral, could we leave the graviton out of the integration variables? If we did, we would not obtain the graviton propagator. This in turn would not allow the interaction of two masses, which occurs due to one graviton exchange. With the integration over the gravitational field, we obtain Einstein's equation as the equation of motion as well as the causal graviton propagator (Using e^{-iS} in the path integral instead of e^{iS} results in the time-reversed propagator with the $-i\epsilon$ prescription [13].). In section "Lessons of Quantum General Relativity," we will see explicit examples of how further classical physics emerges from the path integral. So our starting point for the other fields also points to the quantum nature of the graviton. It is worthwhile to note although we often refer to the classical limit as $\hbar \to 0$, in fact, $\hbar$ is a fixed number (here often set equal to unity) and classical physics emerges in the appropriate kinematic regions. It is not that we have classical physics and then treat quantization as an optional extra step. Rather, the modern view is that our starting point is quantum and that the understanding of the classical limit is the extra step.

我们也借助引言中提到的路径积分对该问题进行了部分探讨。度规出现在拉格朗日量中，是因为我们需要描述弯曲空间中的粒子。既然其余相互作用都已由路径积分描述，我们能否将引力子排除在积分变量之外？如果我们这么做，就无法得到引力子传播子，进而也无法描述两个质量之间因交换单个引力子产生的相互作用。对引力场积分后，我们可以得到爱因斯坦方程作为运动方程，同时得到因果性的引力子传播子 (在路径积分中使用 e^{-iS} 而非 e^{iS} ，会得到遵循 $-i\epsilon$ 规则的时间反转传播子 [13]。)。在“量子广义相对论的启示”一节中，我们会通过具体实例说明经典物理如何进一步从路径积分中涌现。因此，我们对其他场的出发点同样指向引力子的量子属性。值得注意的是，尽管我们常将经典极限称为 $\hbar \to 0$ ，但实际上 $\hbar$ 是一个固定数值 (此处通常设为 1)，经典物理是在合适的运动学区域中涌现出来的。并非先有经典物理，再将量子化作为一个可选的额外步骤。恰恰相反，现代观点认为我们的出发点是量子理论，理解经典极限才是额外的步骤。

Detour into QED

量子电动力学迂回讲解

The effective field theory treatment is not a change in quantum field theory. It is rather using regular QFT with a careful attention to the scales in the problem. The common features of EFT are also seen in other theories in certain limits. In other publications, I have used the sigma model to motivate EFT techniques [2-5], and indeed, that analogy is very useful for gravity. In order to here give a different example, we can use QED for this purpose.

有效场论的处理方式并非对量子场论的修改，而是运用正则量子场论，同时密切关注问题中的能量标度。有效场论的共性在其他理论的特定极限下也会显现。我在其他发表成果中曾用 σ 模型阐释有效场论方法 [2-5]，这个类比确实对引力研究非常有用。为了在这里给出一个不同的示例，我们可以用量子电动力学达成这个目的。

The QED path integral is given by

量子电动力学的路径积分表述如下

$$Z[J] = \int \left[dA_\mu d\psi\right]_\Lambda e^{i\int d^4x[\mathcal{L}_{QED}(A,\psi)-J_\mu A^\mu]} \tag{6}$$

The subscript Λ , implying a limited range of this theory, is also appropriate here. This is because we know that the photon is not the correct degree of freedom at all energies. Above the scale of electroweak symmetry breaking, it is replaced by linear combinations of the $SU(2)_L$ gauge field W_μ^3 , the hypercharge field B_μ , and the Higgs boson. Treating the photon as a separate field is only valid below the electroweak scale. But we do not need to know that fact for QED to work at low energies. Similar comments apply to the charged fermion here. At high energy, the fermion mass eigenstates are decomposed into different fields (the weak eigenstates) and also involve the Higgs field.

下标 Λ 代表该理论的适用范围有限，放在这里也十分恰当。这是因为我们知道，光子并非在所有能量下都是正确的自由度。高于电弱对称破缺能标时，它会被替换为 $SU(2)_L$ 规范场 W_μ^3 、超荷场 B_μ 和希格斯玻色子的线性组合。将光子视为独立场仅在电弱能标以下成立。但量子电动力学要在低能区生效，我们并不需要知道这个事实。这里的带电费米子也遵循类似的逻辑。高能下，费米子质量本征态会分解为不同的场 (弱本征态)，还会涉及希格斯场。

We can explore this at even lower energies - below the mass of the fermion. In this case, in situations where the external fermions are not present, they still propagate in loops in the original theory but can be removed from the effective field theory for photons. That is, we can form an effective action for the photons by integrating out the massive fermion. We do this by performing the path integral over the fermion field to define an effective Lagrangian involving only the photon

我们可以在更低的能量区间——费米子质量以下——研究这个问题。在这种情况下，如果不存在外部费米子，它们仍然会在原理论的圈图中传播，但可以从光子的有效场论中剔除。也就是说，我们可以通过积掉质量费米子得到光子的有效作用量。我们对费米子场做路径积分，就可以得到仅包含光子的有效拉格朗日量

$$e^{i\int d^4x\mathcal{L}_{eff}(A)} = \int \left[d\psi\right]_\Lambda e^{i\int d^4x[\mathcal{L}_{QED}(A,\psi)}. \tag{7}$$

This leaves behind the effective field theory defined by

这就得到了由下式定义的有效场论

$$Z[J] = \int [dA_\mu]_m e^{i\int d^4x \left[\mathcal{L}_{eff}(A) - J_\mu A^\mu\right]}. \tag{8}$$

The subscript on the path integration is now the mass m rather than the electroweak scale, because the effective field theory is only valid below that mass.

现在路径积分的下标是质量 m 而非电弱能标，因为有效场论仅在该质量以下成立。

In practice, we can do this in perturbation theory by matching the full theory to the effective theory. At leading order in the electric charge, this involves the vacuum polarization diagram, $\Pi_{\mu\nu} i(q) = \left(q_\mu q_\upsilon - \eta_{\mu\upsilon} q^2\right) \Pi\left(q^2\right)$, which is described in momentum space using dimensional regularization

实际操作中，我们可以在微扰论框架下通过将全理论与有效理论匹配完成这个过程。在电荷的领头阶，这涉及真空极化图 $\Pi_{\mu\nu} i(q) = \left(q_\mu q_\upsilon - \eta_{\mu\upsilon} q^2\right) \Pi\left(q^2\right)$ ，该过程在动量空间用维数正规化描述

$$\begin{aligned}\Pi\left(q^2\right) &= \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + \log 4\pi - \gamma - \log\frac{m^2}{\mu^2} - \frac{q^2}{5m^2}\right]\left(q^2 << m^2\right) \\ &= \frac{\alpha}{3\pi}\left[\frac{1}{\varepsilon} + \log 4\pi - \gamma - \log\frac{-q^2}{\mu^2}\right]\left(q^2 >> m^2\right)\end{aligned} \tag{9}$$

where $\varepsilon = (d-4)/2$. At low energy, only the top line is relevant. We know what to do with the divergence - it goes into the renormalization of the electric charge. However, we should also note the $\log m^2$ term. This is present even for the heaviest masses. However, if we are to measure the electric charge at $q^2 = 0$, it also goes into the definition of the charge. The residual describes the deviation from the result at $q^2 = 0$,

其中 $\varepsilon = (d-4)/2$ 。低能下只有第一行是相关的。我们知道该怎么处理发散——发散会被吸收到电荷的重整化中。不过我们也需要注意 $\log m^2$ 项。即使是最重的质量，这个项也依然存在。但如果我们要在 $q^2 = 0$ 处测量电荷，这个项也会被纳入电荷的定义中。剩余部分描述了对 $q^2 = 0$ 处结果的偏离，

$$\hat{\Pi}_{\mu\nu}(q) = \Pi_{\mu\nu}(q) - \Pi_{\mu\nu}(0) = \frac{\alpha}{15\pi}\left(q_\mu q_\nu - \eta_{\mu\nu} q^2\right)\frac{q^2}{m^2}\ \left(q^2 << m^2\right). \tag{10}$$

This result can be described by an effective Lagrangian containing only the photon field (Here, I have chosen the $1/4e^2$ normalization to underscore that the renormalization of the electric charge is applied at $q^2 = 0$.)

这个结果可以用仅含光子场的有效拉格朗日量描述 (此处我选用 $1/4e^2$ 归一化，以强调电荷重整化是在 $q^2 = 0$ 处进行的。)

$$\mathcal{L}_{eff}(A) = -\frac{1}{4e^2(0)} F_{\mu\upsilon} F^{\mu\upsilon} - \frac{1}{240\pi^2 m^2} F_{\mu\upsilon} \Box F^{\mu\upsilon}. \tag{11}$$

The form of this Lagrangian has been chosen to match with the vacuum polarization amplitude when a matrix element is taken.

我们将拉格朗日量取为这种形式，是为了在计算矩阵元时匹配真空极化振幅。

The most important point here is that the new term in the effective Lagrangian is local. This follows from the uncertainty principle. Effects from high energy/momentum appear only at short distance in coordinate space. Such effects can be Taylor expanded in the light momenta and are then represented by a local derivative expansion for the effective Lagrangian. The takeaway is that effects from high energy appear local when viewed at low energy and that they can be represented by local Lagrangians.

这里最重要的一点是，有效拉格朗日量中的新项是定域项。这可以由不确定原理导出: 高能/动量效应仅在坐标空间的短距离处显现。这类效应可以对光动量做泰勒展开，最终表示为有效拉格朗日量的定域导数展开。结论就是: 低能区观测到的高能效应是定域的，可以用定域拉格朗日量描述。

The other important property demonstrated here is the decoupling of the heavy mass. You can see through Eq. 9 that the vacuum polarization depends on the logarithm of the heavy mass. However, this is absorbed into the definition of the renormalized coupling. In this sense, the electric charge depends on the masses of charged particles, no matter how heavy. However, there is no physics in this dependence - we only use the measured value of the charge. This is the Appelquist-Carazzone theorem at work [20]. The effects of a heavy particle either appear in the renormalization of the coupling constants of the theory or are suppressed by powers of the heavy mass (An exception is when integrating out the heavy particle violates the symmetry of the theory.).

此处展示的另一重要性质是大质量退耦。你可以通过式 (9) 看出，真空极化依赖于大质量的对数。但这一点会被吸收进重整化耦合的定义中。从这个意义上说，电荷依赖于带电粒子的质量，无论粒子质量多大。不过这种依赖不包含任何新物理——我们只使用电荷的测量值。这就是阿佩尔奎斯特-卡拉佐内定理的体现 [20]。重粒子的效应要么出现在理论耦合常数的重整化中，要么被大质量的幂次压低 (唯一例外是积去重粒子后破坏了理论对称性的情况)。

This is perhaps a good place to note that despite our early discussion which emphasized that we treat the path integral as correct below some scale Λ , we generally do not use a cutoff in calculations. Dimensional regularization respects the symmetries of many theories where a cutoff often does not, and it is easy to use. However, the loop integration does run over all scales including those beyond the applicability of the effective field theory. This is nevertheless acceptable. The ”wrong” behavior at high energy appears as a local effect and satisfies the Appelquist-Carazzone theorem. It then disappears into the renormalization and identification of the parameters of the local effective Lagrangian. Those parameters will be the appropriate ones as long as the EFT is not applied outside of its range of validity.

这里或许适合说明一点: 尽管我们之前的讨论强调，我们将路径积分视为在某一能标 Λ 以下正确，但我们在计算中通常不使用截断。维数正则化能够满足许多理论的对称性，而截断往往做不到这一点，且维数正则化使用方便。不过圈积分确实会覆盖所有能标，包括超出有效场论适用范围的能标，但这依然是可接受的。高能区的“错误”表现表现为局域效应，满足阿佩尔奎斯特-卡拉佐内定理，之后它会被吸收进局域有效拉格朗日量参数的重整化和确定过程中。只要我们不在有效场论的适用范围之外使用它，这些参数就是合适的。

Another feature that can be seen here is that there is no Ostrogradsky instability [21, 22] associated with the higher derivatives. This refers to the result in classical mechanics where in theories with higher time derivatives the Hamiltonian, calculated by canonical methods, exhibits an instability. While the second term has the higher time derivatives, in practice, it does not lead to any instability. At low energy, its effect is small compared to the usual energy. The higher derivatives can become comparable and potentially trigger an instability, only at energies which are far higher than those appropriate for the effective field theory [23]. Despite the extra derivatives, the classical limit of the effective field theory is usual E&M.

我们在此还能看到另一个特点: 高阶导数不会带来奥斯特罗格拉德斯基不稳定性 [21, 22] 。经典力学中有这样一个结论: 在含高阶时间导数的理论中，通过正则方法得到的哈密顿量会存在不稳定性。尽管我们这里的第二项含有高阶时间导数，但实际上它不会引发任何不稳定性。在低能区，它的效应比常规能量效应要小。只有在远高于有效场论适用能标的区域，高阶导数的效应才会变得可与常规效应比拟，才有可能引发不稳定性 [23]。哪怕多了额外导数，有效场论的经典极限依然是常规的电磁学。

There are corrections to this result suppressed by more powers of the mass. The next term in the effective Lagrangian is that of Euler and Heisenberg

这个结果还存在被更高质量幂次压低的修正项。有效拉格朗日量的下一项就是欧拉-海森堡项

$$\mathcal{L}_{EH} = \frac{\alpha^2}{90m^4}\left[\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \frac{7}{4}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2\right] \tag{12}$$

which occurs due to the box diagram. This mediates interactions of photons.

它由箱图产生，介导光子之间的相互作用。

It is also instructive to look at the opposite extreme, where the mass of the fermion goes to zero or the relative momentum transfer is large compared to the mass. In this case, there can be no expansion in inverse powers of the mass. In the vacuum polarization diagram, the logarithm becomes more important. This is nonanalytic and cannot be Taylor expanded in derivatives. If we try to represent it in position space, it would be non-local, represented by the non-local function

我们考察另一种极端情况也很有启发: 费米子质量趋于零，或者相对动量传递远大于质量。在这种情况下，我们无法对质量的逆幂做展开。在真空极化图中，对数项变得更重要，它是非解析的，无法对导数做泰勒展开。如果我们尝试在位置空间表示它，它会是非定域的，可由非定域函数表示

$$\langle x|\log\Box|y\rangle \equiv \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot(x-y)} \log\left(-q^2\right) \tag{13}$$

Physically, this is non-local because massless fields can propagate long distances. If we were to try to match this to an effective action, it would also have to be non-local, schematically represented by [24]

物理上，它是非定域的，因为无质量场可以长距离传播。如果我们尝试将它匹配到有效作用量，有效作用量也必须是非定域的，形式上可表示为 [24]

$$S_{\text{light}} = \int d^4x - \frac{1}{4}F_{\mu\nu}\left[\frac{1}{e^2(\mu)} - \frac{1}{12\pi^2}\log\frac{\Box}{\mu^2}\right]F^{\mu\nu}, \tag{14}$$

with the shorthand notation

使用简写记法为

$$\int d^4xA\log\Box B \equiv \int d^4xd^4yA\left(x\right)\langle x\left|\log\Box\right| y\rangle B\left(y\right). \tag{15}$$

In Eq. 14, we note the appearance of the running coupling constant. However, the main point here is that massless fields yield non-analytic terms such as $\log q^2$ in momentum space and non-localities in position space.

我们注意到，式 (14) 中出现了跑动耦合常数。不过这里的核心点是: 无质量场会在动量空间产生 $\log q^2$ 这类非解析项，在位置空间产生非定域性。

The effective field theory is a full quantum field theory. This can be seen by the fact that the effective action, Eq. 11, still contains the integration over the photon field.

有效场论是完整的量子场论，这点可以从式 (11) 的有效作用量仍包含对光子场的积分看出。

General Relativity as an Effective Field Theory

作为有效场论的广义相对论

While most pedagogic treatments of general relativity emphasize geometry and curved spacetime, it is also possible to develop it as a gauge field theory [5, 25, 26]. If we want to obtain a field theory coupled to energy and momentum, we will gauge spacetime translations, which are the corresponding symmetries. This leads to general covariance, the metric as a dynamical field and covariant derivatives. The action then must be an invariant, with the simplest terms being the cosmological constant and the Einstein action. The geometric treatment is exceptionally powerful for the classical theory. The field theory treatment is conceptually closer to the development of the Standard Model and is more useful for the quantum theory. Effective field theory then helps make sense of the quantum field theory.

大多数广义相对论的教学内容都强调几何与弯曲时空，但我们也可以将其发展为规范场论 [5, 25, 26]。如果我们想要得到一个耦合能量与动量的场论，就需要对作为对应对称性的时空平移做规范处理，这会得到广义协变性、作为动力学场的度规以及协变导数。此时作用量必须是不变量，最简单的项就是宇宙学常数项与爱因斯坦作用量。几何处理对于经典理论格外有力，而场论处理在概念上更接近标准模型的发展，对量子理论也更有用。有效场论则可以帮助我们理解量子场论的意义。

The ultimate origin of the gravitational interactions may not be known, but we do know the symmetry of the theory - general covariance. The unknown physics from high energy will produce a local Lagrangian at low energy. The curvatures are second order in derivatives of the metric, so the action can be ordered in the derivative/energy expansion, with the first several terms being

我们或许还不清楚引力相互作用的终极起源，但我们确定该理论的对称性——广义协变性。来自高能的未知物理会在低能区给出定域拉格朗日量, 曲率是度规的二阶导数, 因此作用量可以按导数/能量展开排序，前几项为

$$S_{\text{grav}} = \int d^4x\sqrt{-g}\left[-\Lambda + \frac{2}{\kappa^2}R + c_1R^2 + c_2R_{\mu\nu}R^{\mu\nu} + \ldots\right] \tag{16}$$

with Λ being the cosmological constant and $\kappa^2 = 32\pi G$. The effective field theory by itself says nothing about the magnitude of the various constants. If we do not know the underlying theory, we need to measure these parameters. At ordinary energies, the effects of the curvature squared terms are negligible if the constants c_1 and c_2 have any normal size (In particular, there would only be noticeable effects in the gravitational interaction at a millimeter if the coefficients were greater than about 10^{65} . If the derivative expansion of general relativity is scaled by the Planck mass, we would expect these dimensionless coefficients to be of order unity. But, given the unexpectedly small value of the cosmological constant and the possibility of new physics as we increase the energy, we should be open to the possibility that this latter expectation is not correct.). Moreover, these do not trigger an Ostrogradsky instability when used as effective interactions [23] in the same way that we saw in the QED example above.

其中 Λ 是宇宙学常数，$\kappa^2 = 32\pi G$ 。有效场论本身并未说明各常数的大小，如果我们不知道基础理论，就需要测量这些参数。在普通能量下，若常数 c_1 和 c_2 为任意常规大小，曲率平方项的效应都可以忽略 (特别是，只有当系数大于约 10^{65} 时，才会在毫米尺度的引力相互作用中出现可观测效应。如果广义相对论的导数展开以普朗克质量标度，我们会预期这些无量纲系数为一阶大小，但考虑到宇宙学常数出乎意料的小，且能量升高时可能存在新物理，我们应当接受上述预期不一定成立的可能性)。此外，将这些项作为有效相互作用使用时，不会像我们上文 QED 例子中那样触发奥斯特罗格拉德斯基不稳定性 [23]。

Including extra terms in the action beyond the Einstein term is not a big deal by itself. But doing this allows one to perform renormalization of the quantum theory. By using the most general Lagrangian consistent with general covariance, we can be sure that all the UV divergences can be renormalized into the various coefficients as long as we use a regularization which does not break general covariance. The quantum field is the fluctuation in the metric which deviates from a given background metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}$. There is a residual covariance associated with the background field. The divergences are at short distance, which by the equivalence principle can be treated as almost flat, so that we know the behavior of propagators at short distance. The one-loop renormalization was carried out beautifully by 't Hooft and Veltman [27]. The divergences due to graviton loops can be represented by an effective Lagrangian of the form

在作用量中加入爱因斯坦项之外的额外项本身并不是什么大问题，但这么做可以让我们完成量子理论的重整化。只要我们使用不破坏广义协变性的正则化，通过使用满足广义协变性的最一般拉格朗日量，我们就可以确定所有紫外发散都能被重整化到各个系数中。量子场是度规偏离给定背景度规 $g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}$ 的涨落，背景场还残余协变性。发散出现在短距离区域，根据等效原理，该区域可以近似为平直时空，因此我们可以知道短距离下传播子的行为。't Hooft 和 Veltman 出色地完成了单圈重整化工作 [27]，引力子圈导致的发散可以表示为如下形式的有效拉格朗日量

$$\mathcal{L}_{\text{div}} = \frac{1}{16\pi^2}\frac{1}{\varepsilon}\left(\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right) \tag{17}$$

while those for other matter fields are similar but with different coefficients. One can see that these divergences can be absorbed into the renormalized values of the coefficients c_1 and c_2 . Power counting in powers of G [5] reveals that higher-order graviton loops yield divergences at higher order in the derivative expansion, i.e., two-loop divergences are of order R^3 [28]. In contrast, higher-order loops of matter fields from renormalizable field theories remain at order R^2 . This also can be seen by power counting, because the divergences in such theories do not have any inverse powers of the mass needed to compensate for extra powers of the derivatives in the numerator.

其他物质场的发散形式类似，但系数不同。可以看到这些发散可以被吸收到系数 c_1 和 c_2 的重整化值中。对 G 幂次做功率计数 [5] 可以发现，更高阶引力子圈会在导数展开的更高阶产生发散，即双圈发散是 R^3 阶 [28]。与之相比，可重整场论中更高阶物质场圈仍保持为 R^2 阶，这也可以通过功率计数得到，因为这类理论中的发散不存在质量的逆幂次来补偿分子中额外的导数幂次。

However, the renormalization of divergences is also not that big of a deal, although it was the focus of this subject for many years. The divergences themselves come from the high-energy end of the theory, which we know is not reliable. The ultimate UV completion will eventually tell us the correct way to treat this domain and will predict the value of the coefficients. So renormalization is a necessary step, but one without much content.

然而，尽管发散的重整化是该领域多年来的研究焦点，它本身也并没有那么重要。发散本身来自理论的高能端，我们知道该区域的理论并不可靠。最终的紫外完备化会告诉我们处理这个区域的正确方法，并给出系数的预言值。因此重整化只是一个必要步骤，本身没有太多实质内容。

The real power of the effective field theory is that it shifts our attention from the UV (where we do not know the physics) to the IR (where we do). There, EFT techniques allow one to make real predictions. This is because we know the light degrees of freedom active there and we know their interactions. At a given order in the energy expansion, we have a small number of coefficients, such as Λ, G, c_1 , and c_2 , so that we have reduced our ignorance of the full theory of quantum gravity to a few constants. However, there are dynamical effects which are independent of these coefficients. This comes from the fact that massless fields like the graviton can propagate large distances, so that this propagation is distinct from any term in a local Lagrangian.

有效场论的真正优势在于，它将我们的关注点从我们尚不了解物理规律的紫外区域，转移到了我们已知规律的红外区域。在红外区域，有效场论技术可以让我们做出切实的预言。这是因为我们清楚该区域活跃的轻自由度，也了解它们的相互作用。在能量展开的任意给定阶数下，我们只需处理少量系数，例如 Λ, G, c_1 和 c_2 ，因此我们将对完整量子引力理论的未知性简化为仅少数几个常数。不过，存在一些不依赖这些系数的动力学效应。这源于引力子这类无质量场可以长距离传播，这种传播特性不同于局域拉格朗日量中任何一项的性质。

As an example, let us display what happens with the gravitational interaction of non-relativistic particles, which will be discussed with more specificity in the next section. At tree level, one graviton exchange gives an amplitude which behaves as $1/q^2$ much like photon exchange in QED. When Fourier transformed, this gives the Newtonian potential. At one-loop level, the amplitude picks up non-analytic behavior from the loops of gravitons. Schematically at the next order in G , we see that this has the form

举个例子，我们来看非相对论粒子引力相互作用的情况，下一节会对它做更具体的讨论。在树图阶，单个引力子交换给出的振幅行为和量子电动力学中的光子交换类似，形式为 $1/q^2$ 。做傅里叶变换后，就得到牛顿势。在单圈阶，引力子圈会给振幅带来非解析行为。粗略来看，在 G 的下一阶，该形式可写为

$$\mathcal{M} \sim \frac{GMm}{q^2}\left[1 + aG\sqrt{-m^2q^2} + bGq^2\log\left(-q^2\right) + cGq^2 + \dots\right] \tag{18}$$

with a, b , and c being constants to be calculated. The analytic term cGq^2 will in practice contain effects from the coefficients c_1 and c_2 , which come from the extra derivatives when we consider the squares of curvatures in the local action. However, the non-analytic terms in this matrix element will be independent of these parameters. If we are to Fourier transform the matrix element to obtain a position space potential, it will have the form

其中 a, b 和 c 是待计算的常数。实际中，解析项 cGq^2 会包含来自系数 c_1 和 c_2 的效应，这两个系数来源于我们对局域作用量中曲率平方项考虑额外导数时产生的贡献。不过，该矩阵元中的非解析项不依赖这些参数。如果我们对矩阵元做傅里叶变换得到位置空间的势，它会具有如下形式

$$V(r) = -\frac{GMm}{r}\left[1 + a'\frac{GM}{r} + b'\frac{G\hbar}{r^2}\right] + c'G^2\delta^3(x). \tag{19}$$

The power law corrections come from the non-analytic terms in momentum space and are independent of the coefficients c_1 and c_2 . When we restore powers of $\hbar$, we can see that the first correction $\sim GM/r$ is classical and the second one $\sim G\hbar/r^2$ is a quantum correction.

幂律修正来自动量空间的非解析项，且不依赖系数 c_1 和 c_2 。当我们恢复 $\hbar$ 的幂次后，可以看到一阶修正 $\sim GM/r$ 是经典修正，二阶修正 $\sim G\hbar/r^2$ 是量子修正。

The form of the low-energy dynamics is similar to what happens in other quantum field theories. One does not need to know what happens at extremely high energies in order to make predictions at ordinary energies - this is the basic message of effective field theory. Knowledge of the low-energy particles and interactions are sufficient. General relativity fits beautifully into this paradigm.

低能动力学的这种形式和其他量子场论中的情况类似。我们不需要知道极高能下发生的物理，就能在普通能标下做出预言——这就是有效场论的核心思想。只要知道低能粒子和它们的相互作用就足够了。广义相对论完美契合这一范式。

Lessons of Quantum General Relativity

量子广义相对论的经验教训

Perturbative quantum field theory is best at calculating transitions and scattering amplitudes. From these, we can learn some features about low-energy quantum gravity. We have waited a long time for a quantum theory of general relativity. While the EFT does not answer all of our questions, let us see what we can do with it.

微扰量子场论最擅长计算跃迁和散射振幅。我们可以从中了解低能量子引力的一些特征。我们为广义相对论的量子理论已经等待了很长时间。虽然有效场论 (EFT) 无法回答我们所有的问题，不妨来看看我们能借助它得到什么。

There Is a Universal Quantum Correction to the Non-relativistic Potential

非相对论势存在普适量子修正

We can calculate the gravitational scattering amplitude for two massive particles at one-loop order. Because all diagrams are included, this is the complete quantum amplitude, and it is a gauge invariant. While the full amplitude is a function of all the kinematic variables, we can take the non-relativistic limit in which the only important variable is the three-momentum transfer $-q^2 = \mathbf{q}^2$. For display purposes, one can then Fourier transform this function to obtain a position space potential. As explained in the previous section, the power law corrections in r follow from the non-analytic momentum dependence and are independent of any divergences or unknown coefficients. The result for two particles of mass M and m is [29, 30]

我们可以计算两个大质量粒子在单圈阶的引力散射振幅。由于包含了所有费曼图，这是完整的量子振幅，且满足规范不变性。尽管完整振幅是所有运动学变量的函数，但我们可以取非相对论极限，该极限下唯一重要的变量是三动量转移 $-q^2 = \mathbf{q}^2$ 。为了方便展示，我们可以对这个函数做傅里叶变换得到位置空间势。如前一节所述，r 中的幂律修正来源于非解析动量依赖，且与任何发散或未知系数无关。质量分别为 M 和 m 的两个粒子的结果为 [29, 30]

$$V(r) = -\frac{GMm}{r}\left[1 + 3\frac{G(M+m)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]. \tag{20}$$

It is obviously the third term which is the quantum correction.

显然第三项就是量子修正项。

It is interesting that result has now been calculated using three methods. The original calculations used the usual Feynman diagram methods. However, the same result can be obtained by modern unitarity-based methods [31, 32]. Here, only the on-shell gravitational Compton amplitudes (those involving two on-shell gravitons) are required. These are related to the unitarity cut in the crossed channel. By evaluating this cut and mapping it onto the cuts of the master Feynman integrals, one can obtain the final result more simply. In addition, because of the property that the graviton amplitude is related to the square of a gauge theory amplitude (the gauge-gravity correspondence or ”double copy” [33, 34]), one really only has to evaluate the QED Compton amplitude in this way of calculating [35]. This avoids needing to use the very messy triple-graviton vertex. Because in this method only on-shell physical gravitons are used, there are no Faddeev-Popov ghosts involved. The original Feynman diagram calculation was done in harmonic (de Donder) gauge, while the on-shell results use a form of an axial gauge, confirming the gauge invariance of the result. The result has also been obtained via dispersion relations [31], where the spectral function also is calculated from the cut, in both harmonic gauge and using the double copy axial gauge.

有趣的是，该结果目前已经通过三种方法计算得到。最初的计算采用常规费曼图方法。但也可以通过现代基于幺正性的方法得到相同结果 [31, 32]。这类方法仅需要壳引力康普顿振幅 (即包含两个在壳引力子的振幅)。这些振幅与交叉道中的幺正切割相关。通过计算该切割并将其映射到主费曼积分的切割上，我们可以更简便地得到最终结果。此外，由于引力子振幅与规范理论振幅的平方相关 (规范引力对应，即“双拷贝”[33, 34])，在这种计算框架下我们实际上只需要计算量子电动力学的康普顿振幅 [35]，这避免了处理十分繁杂的三引力子顶点。由于该方法仅使用在壳物理引力子，不涉及法捷耶夫-波波夫鬼。最初的费曼图计算在调和 (德唐德) 规范下完成，而在壳结果采用了一类轴规范，两种计算都验证了结果的规范不变性。该结果也通过色散关系得到过 [31]，在色散关系方法中，谱函数同样由切割计算得到，且分别在调和规范和双拷贝轴规范下完成了验证。

Another interesting feature of this result is that it is universal. The universality was first found by Holstein and Ross [36] by redoing the Feynman diagram calculations for particles of different spins. In such a method, the universality is remarkable because different Feynman diagrams are involved in the various cases, and the universality is only seen when adding all diagrams together. However, by the unitarity and dispersive methods, the result can be understood to be the consequence of tree-level soft theorems. Electromagnetic Compton amplitudes and gravitational Compton amplitudes are universal in the limit of small momenta [37-39]. This applies not only to elementary particles but to composite macroscopic objects. In the unitary and dispersive methods, the non-analytic terms arise from multiplying together the universal tree amplitudes. The result is then a one-loop soft theorem for quantum gravity.

该结果的另一个有趣特性是它的普适性。霍尔斯坦和罗斯最早通过重新计算不同自旋粒子的费曼图发现了这种普适性 [36]。在这类方法中，普适性十分引人注目，因为不同情形涉及不同的费曼图，只有将所有费曼图相加后才能得到普适结果。但通过幺正性方法和色散方法可以看出，该结果是树级软定理的推论。电磁康普顿振幅和引力康普顿振幅在小动量极限下都是普适的 [37-39]，这不仅适用于基本粒子，也适用于复合宏观物体。在幺正性方法和色散方法中，非解析项来源于普适树振幅的乘积，因此该结果本质上是量子引力的单圈软定理。

Both Classical and Quantum Effects Come from Loop Diagrams

经典效应与量子效应均来自圈图

We are often told that the loop expansion is an expansion in $\hbar$. If this were the case, we would not expect to obtain the classical correction seen in Eq. 20 from one-loop Feynman diagrams. However, the folk theorem is in fact not true. For the gravitational interaction, this has been known since the work of Ishikawa [40] and Gupta and Radford [41], but the insight is more general [42]. At a technical level, when one is counting powers of $\hbar$ by pulling out overall factors of this quantity, there are residual factors left behind. For example, in the Dirac equation, one has $\hbar\bar{\psi}\,(i\partial\!\!\!/ - m/\hbar)\,\psi$. The $m/\hbar$ factor can compensate an overall factor of $\hbar$, as in $G\sqrt{q^2m^2} = G\hbar\sqrt{q^2m^2/\hbar^2}$. On a more philosophical level, if we are to reconstruct the world, including the classical limit, from the path integral treatment, then the classical results need to be contained somewhere in the Feynman diagram expansion.

> 我们常被告知圈展开是对 $\hbar$ 的展开。如果真是这样，我们就不可能从单圈费曼图得到式 (20) 中的经典修正。然而这个普遍说法实际上并不正确。就引力相互作用而言，早在石川 (Ishikawa)[40] 以及古普塔和雷德福德 (Gupta and Radford)[41] 的工作中就已经发现了这一点，但该结论更具普适性 [42]。在技术层面上，当我们通过提取整体的 $\hbar$ 因子来统计 $\hbar$ 的幂次时，总会剩余额外因子。例如狄拉克方程中存在 $\hbar\bar{\psi}(i\partial\!\!\!/ - m/\hbar)\psi$，其中 $m/\hbar$ 因子可以抵消整体的 $\hbar$ 因子，正如 $G\sqrt{q^2m^2} = G\hbar\sqrt{q^2m^2/\hbar^2}$ 所示。从更根本的层面来说，如果我们要从路径积分表述重构包含经典极限在内的整个世界，那么经典结果必然包含在费曼图展开的某处。

This insight was first developed into a calculational program for classical gravitational wave physics by Goldberger and Rothstein [43]. With some further developments, it has now become a subfield for obtaining classical results from QFT techniques. For recent reviews, see [44-46]

> 这一观点首先由戈德伯格和罗思斯坦 (Goldberger and Rothstein)[43] 发展为经典引力波物理的计算方案。经过后续发展，它如今已经成为一个利用量子场论技术获取经典结果的子领域。近期综述参见 [44-46]

There Is No ”Test Mass” Limit for Quantum Effects

量子效应不存在“试验质量”极限

It is common to imagine a test mass of vanishing size moving along a geodesic without itself distorting the spacetime. Perhaps surprisingly, this does not work for quantum effects. The quantum effects sample more than just the geodesic even for small masses. The test mass limit for the classical interaction can be seen in the nonrelativistic potential, Eq. 20, where the classical correction depends on $M + m$. If m is the smaller mass of the two, then sending m to zero leaves just the larger mass determining the correction. But the quantum effect is independent of the relative masses, and both masses contribute equally.

> 人们通常认为，尺寸可忽略的试验质量沿测地线运动，其本身不会扭曲时空。令人惊讶的是，这一结论不适用于量子效应。即便对小质量而言，量子效应也不止作用于测地线。经典相互作用的试验质量极限可以从非相对论势即式 (20) 中看出，经典修正依赖于 $M + m$ 。若 m 是两个质量中较小的那个，将 m 取零后，修正就仅由较大质量决定。但量子效应与相对质量无关，两个质量的贡献相等。

To see this in more detail, let us look at two of the diagrams which contribute to the corrections of the Newtonian potential, shown in Fig. 1. These diagrams contribute to both classical and quantum effects. For the classical portion, Fig. 1a gives a correction which is proportional to M , and that of Fig. 1b yields one proportional to m . So in the test mass limit, Fig. 1b is negligible, and it is reasonable to interpret that of Fig. 1a as a correction to the metric that the test body moves in. However, for the quantum effects, both diagrams 1a and 1b give equal quantum corrections, so the idea of a test mass does not work for these diagrams. In Fig. 1b, the graviton propagates for a long distance and samples the gravitational field not only along the geodesic but also at different points. It is an irreducible tidal effect. As a correction to the Newtonian potential, it does not vanish as the mass is taken to zero. There are other diagrams with this property also.

为了更详细地说明这一点，我们来看图 1 中对牛顿势修正有贡献的两个费曼图。这两个费曼图对经典效应和量子效应都有贡献。对于经典部分，图 1a 给出的修正正比于 M，图 1b 给出的修正正比于 m。因此在试验质量极限下，图 1b 的贡献可以忽略，将图 1a 的修正解释为试验物体运动所在度规的修正是合理的。但对于量子效应，图 1a 和图 1b 给出的量子修正相等，因此试验质量的概念不适用于这些费曼图。在图 1b 中，引力子传播长距离，不仅会沿测地线，还会在不同位置取样引力场。这是一种不可约的潮汐效应。作为对牛顿势的修正，它不会随质量取零而消失。还有其他费曼图也具备这一性质。

The Bending of Massless Particles Is Not Universal

无质量粒子的偏折不具有普适性

One can address the bending of light also by a scattering calculation. This should not be done by calculating the quantum cross-section and then using the classical relation to the bending angle - that procedure only works for $1/r$ potentials. However, the eikonal approximation is designed to recover the geometric optics result at large impact parameter b. By calculating the peak of eikonal phase, one reproduces the classical bending angle to impressively high order in inverse powers of the impact parameter [47,48]. One can add quantum corrections to this result. The use of unitarity methods, where the diagrams are reconstructed from the on-shell cuts, simplifies the calculation greatly.

我们也可以通过散射计算来处理光的偏折。不应通过计算量子截面，再利用它与偏折角的经典关系来完成这个计算——该方法仅对 $1/r$ 势适用。而程函近似本就设计用于在大碰撞参数 b 下还原几何光学结果。通过计算程函相位的峰值，我们可以在碰撞参数倒数的高阶下高精度重现经典偏折角 [47,48]，还可以对该结果添加量子修正。使用从在壳切割重构费曼图的幺正性方法，可以极大简化计算。

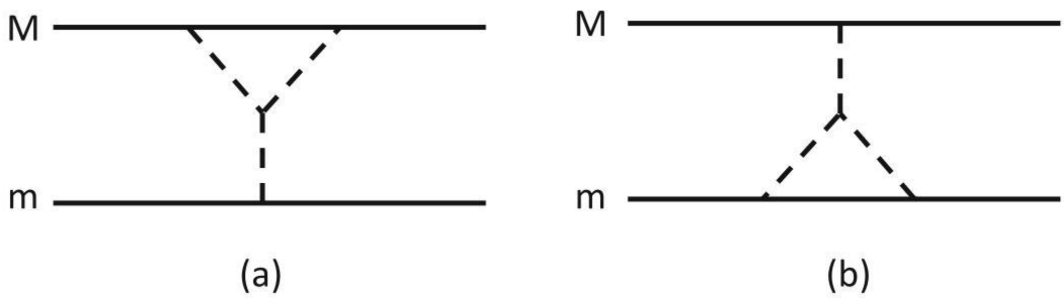

Fig. 1 Two diagrams which contribute to the gravitational scattering of two masses. The solid lines represent particles with a large mass M and a small mass m. The dashed lines are gravitons

图 1 对两个质量的引力散射有贡献的两张图。实线分别代表大质量 M 粒子和小质量 m 粒子，虚线是引力子

As usual, the result is unobservably small at any reasonable impact parameter. However, the interesting aspect is that it is not the same for all massless particles [49-51]. One finds

和通常情况一样，在任何合理的碰撞参数下，该结果都小到无法观测，但有趣的是，不同无质量粒子得到的结果并不相同 [49-51]。我们得到

$$\theta = \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8c_b - 47 + 64\log(2r_0/b)}{\pi}\frac{G^2M\hbar}{b^3} \tag{21}$$

where $c_b = (371/120, 113/120, -29/8)$ for scalars, photons, and gravitons, respectively (The original calculation only included gravitons in the cuts. Subsequent calculations [50, 51] correctly also include massless matter particles.), and r_0 is an infrared cutoff. We should not be surprised at this lack of universality, as there are no low-energy theorems for massless particles as there were in the non-relativistic limit. The tidal effects include the long-distance propagation of gravitons and the massless particles themselves. There are entirely different diagrams involved for the different cases.

其中 $c_b = (371/120, 113/120, -29/8)$ 分别对应标量粒子、光子和引力子 (原始计算仅在切割中包含了引力子，后续计算 [50, 51] 正确加入了无质量物质粒子)，r_0 是红外截断。我们不应对这种非普适性感到惊讶，因为并不存在非相对论极限下那种适用于无质量粒子的低能定理。潮汐效应包含引力子和无质量粒子本身的长程传播，不同情况涉及完全不同的费曼图。

This result has interesting implications for the concepts of light cones and geodesics, as will be discussed below.

该结果对光锥和测地线的概念有重要意义，我们将在下文讨论。

G and Λ Are Not Running Couplings in Physical Processes

G 和 Λ 并非物理过程中的跑动耦合

We are used to having our coupling constants depend on the energy scale at which one measures it. The idea of a running coupling captures the effects of quantum processes relevant for that scale. These are universal because they come along with the renormalization of the couplings. In a mass-independent scheme, the $1/\varepsilon$ in dimensional regularization are always accompanied by $\log(\text{ Energy })^2/\mu^2$, where (Energy) represents some of the kinematic variable in the process under investigation (The use of a mass-independent scheme is useful to avoid confusion with $1/\varepsilon - \log m^2/\mu^2$ which does not indicate kinematic running.).

我们通常认为耦合常数依赖于测量它的能量标度。跑动耦合的概念体现了与该能标相关的量子过程效应。这些效应是普适的，因为它们伴随耦合的重整化出现。在与质量无关的方案中，量纲正则化下的 $1/\varepsilon$ 始终伴随着 $\log(\text{ Energy })^2/\mu^2$ ，其中 (能量) 代表所研究过程中的部分运动学变量 (使用与质量无关的方案有助于避免与 $1/\varepsilon - \log m^2/\mu^2$ 混淆，后者并不代表运动学跑动。)

In the gravitational action, the coefficients quadratic in the curvature, i.e., c_1 and c_2 , obey this paradigm. We have seen that loops of massless particles, including gravitons, renormalize the coefficients of operators of order R^2 . This can be seen most simply in the gravitational vacuum polarization diagram, and here, the divergences do come along with factors $\log q^2/\mu^2$. In analogy with the electromagnetic case in Eq. 14, we can represent this physical effect to an action such as

在引力作用量中，曲率二次项的系数，即 c_1 和 c_2，符合这一范式。我们已经看到，包括引力子在内的无质量粒子圈图，会重整化 R^2 阶算符的系数。这一点在引力真空极化图中最容易看出，在此处发散的确伴随因子 $\log q^2/\mu^2$。类比式 (14) 中的电磁情况，我们可以将该物理效应表示为如下作用量

$$\sim \int d^4x\sqrt{-g}\left[c_1(\mu)R^2 + bR\log\left(\Box/\mu^2\right)R + \ldots\right]. \tag{22}$$

This is exactly what is done in the formalism developed by Barvinsky and Vilkovisky [52-55]. It is straightforward to convert this into a renormalization group equation for c_1.

这正是巴尔文斯基 (Barvinsky) 和维尔科夫斯基 (Vilkovisky) 所发展形式体系中的做法 [52-55]，可以直接由此推导出 c_1 的重整化群方程。

But Λ and G are not like this. When we calculate loops of gravitons or other massless particles using dimensional regularization, Λ and G are not renormalized - only the curvature squared terms are. So the logarithms which accompany renormalization are not present.

但 Λ 和 G 并非如此。当我们用量纲正则化计算引力子或其他无质量粒子的圈图时，Λ 和 G 不会被重整化——只有曲率平方项会被重整化。因此伴随重整化的对数项并不存在。

However, there is renormalization of Λ and G when one has loops of massive particles, although these have a different character. For example, for a scalar of mass m, the one-loop divergence relevant for the cosmological constant is [56]

不过，当存在大质量粒子的圈图时，Λ 和 G 确实会发生重整化，尽管这类重整化具有不同的性质。例如，对于质量为 m 的标量粒子，与宇宙学常数相关的单圈发散为 [56]

$$\delta\Lambda = -\frac{m^4}{32\pi^2}\left[\frac{1}{\varepsilon} - \gamma + \log(4\pi) + \log\frac{\mu^2}{m^2} + \frac{3}{2}\right]. \tag{23}$$

Note that the logarithm here is $\log m^2$. It has nothing to do with the external scales of the problem. Once one measures the cosmological constant at one scale, it does not change when working at a different scale. Following the $\log\mu$ dependence does not signify running in physical reactions when masses are present because $\log m^2/\mu^2$ does not change with scale. Another example of this sort is in the QED vacuum polarization for $q^2 << m^2$, i.e., Eq. 9. At low energy, the $\log m_t^2/\mu^2$ from a top quark in the loop does not lead to running. The reader who would like to see this realized in a EFT setting close to gravity, but with real comparison to experiment, can study the renormalization of the pion decay constant F_π^2 in chiral perturbation theory [57,58], which plays the same role in the chiral EFT as $1/G$ does in general relativity.

注意此处的对数是 $\log m^2$，它和问题的外部标度无关。一旦在某一标度测量了宇宙学常数，它在不同标度下计算时也不会发生变化。当存在质量时，遵循 $\log\mu$ 依赖关系并不意味着物理反应中存在跑动，因为 $\log m^2/\mu^2$ 不随标度变化。这类情况的另一个例子是 $q^2 << m^2$ 的 QED 真空极化，即式 (9)。低能下，圈图中顶夸克带来的 $\log m_t^2/\mu^2$ 不会引发跑动。如果读者想要了解这一点在贴近引力的有效场论框架下的实现，还能和实验进行实际对比，可以研究手征微扰论中 π 介子衰变常数 F_π^2 的重整化 [57,58]，它在手征有效场论中扮演的角色，和 $1/G$ 在广义相对论中扮演的角色完全相同。

One can also make this theoretical argument by noting that the running of these parameters with external scales would need to match on to some effective action, most likely non-local. For logarithmic running, we have seen now how this works at the curvature squared order in gravity, Eq. 22, and in QED, Eq. 14. For the cosmological constant and for the Einstein term, general covariance says that there are no non-local operators which share the form of the local operator. The leading non-local operator closest to the cosmological constant has been calculated [56], but it has a different structure. In particular, in an expansion about flat space, the cosmological constant has a term linear in the gravitational field, while any nonlocal partner must have at least two fields.

我们也可以通过如下理论论证说明这一点: 这些参数随外部标度的跑动需要匹配到某个有效作用量，且该作用量极有可能是非定域的。我们已经知道对于对数跑动，这在引力的曲率平方阶 (式 22) 和量子电动力学 (式 14) 中是如何成立的。对于宇宙学常数和爱因斯坦项，广义协变性要求不存在和定域算符形式相同的非定域算符。最接近宇宙学常数的领头阶非定域算符已经被计算出来 [56]，但它的结构完全不同。具体来说，在平直空间展开下，宇宙学常数含有一个引力场的线性项，而任何非定域配对算符都至少含有两个引力场。

When using a cutoff regularization, one finds divergences depending on powers of the cutoff such as $(\text{cutoff})^4$ or $(\text{cutoff})^2$ for Λ and G (However, some of the naive dependences often quoted are in fact absent due to cutoff-dependent terms in the path integral Jacobean [59].). It needs to be emphasized that these do not define running parameters in physical processes. On the one hand, physical results do not depend on regularization scheme, and the absence of these dependences in dimensional regularization implies that they are not physical. They are absorbed when measuring Λ or G at a given scale and do not change when the external scales do. In an effective action framework, powers of external scales would be represented in the local Lagrangian by powers of derivatives or curvatures. They are already included in the local terms in the derivative expansion of the EFT, but with fixed coefficients. Again, the related case of chiral perturbation theory is an experimentally tested EFT where one verifies these comments.

使用截断正则化时，会发现发散依赖于截断的幂次，例如对于 Λ 和 G，发散为截断的 ^{4} 次幂或截断的 ^{2} 次幂 (然而，由于路径积分雅可比行列式中依赖截断的项，文献中常提及的部分直观依赖实际上并不存在 [59])。需要强调的是，这些发散并不定义物理过程中的跑动参数。一方面，物理结果不依赖于正则化方案，这些依赖在维数正则化中不存在，说明它们并非物理效应。当在给定标度下测量 Λ 或 G 时，这些依赖就被吸收了，外标度改变时它们也不会发生变化。在有效作用量框架中，外标度的幂次会在局域拉格朗日量中由导数或曲率的幂次表示。它们已经被包含在有效场论 (EFT) 导数展开的局域项中，只是系数是固定的。此外，手征微扰论这一相关案例是经过实验检验的有效场论，可以验证上述结论。

Finally, one could just try to identify a running coupling in the calculation of physical processes. Perhaps there turns out to be some useful way to capture the quantum effects as a function of scale. There are multiple calculations which have been done. In any one process, one could identify some energy dependence which describes the quantum effects for that process. But there is no useful or universal identification that works with multiple processes [60].

最后，人们仍可以尝试在物理过程的计算中定义一个跑动耦合。或许确实存在某种实用的方法，可以将量子效应描述为标度的函数。目前已有多项相关计算。在任意单个过程中，我们都可以找出描述该过程量子效应的某种能量依赖关系。但并不存在适用于多个过程的实用且普适的定义方式 [60]。

The subfield of asymptotic safety [61-63] uses a IR cutoff and defines the theory by using Wilsonian ideas for running that cutoff from a UV fixed point down to zero energy. However, this is just used to define the physical theory once the full range of the cutoff is included. It does not mean that the physical parameters run and is not intrinsically in conflict with the above discussion. But it should be recognized that it is incorrect to use that power law cutoff dependence in physical settings as if it were running in phenomenological applications [64]. More modern treatments use the derivative expansion to describe energy behavior in physical processes [65] and are more in line with the effective field theory treatment.

渐近安全子领域 [61-63] 使用红外截断，通过威尔逊的方法定义理论: 让截断从紫外不动点跑动到零能量。但这只是在包含截断的全范围后用来定义物理理论，它并不意味着物理参数是跑动的，也和上述讨论不存在本质冲突。但需要认识到，在唯象学应用中将这种幂律截断依赖当作物理过程中的跑动来使用是错误的 [64]。更现代的处理方法使用导数展开来描述物理过程中的能量行为 [65]，更符合有效场论的处理框架。

There Is No ”Quantum Metric”

不存在“量子度规”

It is tempting to look for quantum effects modifying the metric describing various classical solutions. We do not do this for QED (there is no ”quantum corrected electric field” surrounding a charge), but classical solutions play such a foundational role in general relativity that we are certainly interested in this question.

人们很自然会想要寻找量子效应来修正描述各类经典解的度规。我们在量子电动力学中不会这么做 (电荷周围不存在“量子修正电场”)，但经典解在广义相对论中发挥着基础性作用，因此我们自然会关注这个问题。

For cases where gravity remains classical and the quantum effects are due to matter fields, such as those involving photons in the Reissner-Nordström metric, this question appears well defined [66]. For example, the quantum correction due to photon loops in g_{00} is (In this equation, I have removed the factor of $\hbar$ from the usual definition of α so that $\alpha = e^2/4\pi$.).

对于引力仍为经典，而量子效应来源于物质场的情况，例如赖斯纳-诺德斯特龙度规中包含光子的情况，这个问题的定义是清晰的 [66]。例如，g_{00} 中光子圈带来的量子修正为 (本式中，我从 α 的通常定义中去掉了因子 $\hbar$ ，因此得到 $\alpha = e^2/4\pi$ 。)。

$$g_{00} = 1 - \frac{2GM}{r} + \frac{G\alpha}{r^2} - \frac{8}{3\pi}\frac{G\alpha\hbar}{Mr^3}. \tag{24}$$

Since gravity is classical here, this result just follows from loop corrections to the energy momentum tensor. Here again, loop diagrams reproduce both the leading classical correction and a quantum correction.

由于此处引力是经典的，该结果直接来自能量动量张量的圈修正。在这里，圈图同样同时给出了领头阶经典修正和量子修正。

However, when gravity itself is treated in QFT, the result is more problematic. Part of the problem is that the metric itself is not a well-defined quantum concept. Quantum physics traditionally describes transition amplitudes and the like, and the results described earlier in this section have been derived from these. But the traditional class of well-defined quantum objects does not include the field variable itself.

但如果将引力本身放在量子场论框架下处理，结果就会变得更成问题。部分问题在于，度规本身并不是一个定义良好的量子概念。量子物理传统上描述跃迁振幅等对象，本节前面描述的结果也都是从这些对象推导出来的。而传统上定义良好的量子对象类别并不包含场变量本身。

Along these lines, there have been attempts [67-70] (including one by the present author) to calculate the quantum corrections to the Schwarzschild metric. These have been criticized by Kirilin [71] as not being invariant under the reparametrization of the gravitational field, i.e., the way that one choses to expand about a background metric. In QFT, it is not supposed to make a difference if we perform field redefinitions as long as the identification of the on-shell free fields is maintained. This is commonly used and is referred to as Haag's theorem [72]. However, even in non-gravitational QFT, the theorem only applies to on-shell matrix elements. Off-shell quantities and intermediate results are not similarly invariant. A given set of quantum corrections to a metric are intermediate results to a full calculation and depend not only on the gauge (which is to be expected) but also on the field parameterization (which is harder to overcome).

沿着这一思路，已有多项研究 [67-70](包括本文作者的一项工作) 尝试计算施瓦西度规的量子修正。基里林 [71] 批评这些研究不满足引力重参数化下的不变性，即不满足关于背景度规展开方式选择下的不变性。在量子场论中，只要保持在壳自由场的识别不变，场重定义不应该改变结果，这是一个被广泛应用的结论，称为黑格定理 [72]。然而，即便是在非引力量子场论中，该定理也仅适用于在壳矩阵元。离壳量和中间结果并不具备这种不变性。对度规的一组量子修正属于完整计算中的中间结果，不仅依赖于规范 (这在预料之中)，还依赖于场参数化 (这一点更难解决)。

The semiclassical idea of the expectation value of the metric $\langle g_{\mu\nu} \rangle$ is also not a valid quantum object when the gravitational field itself is quantum. It is subject to Kirilin's criticism. One would need to understand how this expectation value would be measured in a quantum process for it to be well defined.

当引力场本身是量子场时，半经典观点中度量 $\langle g_{\mu\nu} \rangle$ 的期望值也不是一个有效的量子对象。它同样受到基里林的批评。需要先明确如何在量子过程中测量这个期望值，它才能成为定义良好的对象。

This problem as described above is one of perturbation theory. The idea of a covariant non-local effective action seems well defined. If treated fully, this presumably could reveal the nature of quantum solutions. But in practice such actions are only approximately known. The method of Barvinsky and Vilkovisky called the expansion in the curvature [52-54] and referred to briefly in Eq. 22 is an example. It appears expressed in terms of curvatures, so that might be independent of field redefinitions. However, there is not a unique understanding of what is meant by $\log \Box$ – this part of the calculation is not expressed in terms of curvatures. The correction to the logarithmic term in the effective action involves structures which are generically of the form $R^2 \frac{1}{\Box} R$. These are of the same order in both the loop expansion and in the derivative expansion as the logarithmic correction. Dimensional analysis can be used to show that in the Schwarzschild case, near the horizon, these corrections are of the same magnitude as the logarithmic ones. So even here we see some limits of perturbation theory.

上文描述的这个问题属于微扰论框架下的问题。协变非局域有效作用量这一思路本身定义是清晰的。如果完全处理，它或许能够揭示量子解的本质。但实际上这类有效作用量只有近似结果。巴尔文斯基和维尔科夫斯基提出的曲率展开方法 [52-54] 就是一个例子，式 (22) 也简要提到了该方法。它看起来用曲率来表示结果，因此可能独立于场重定义。然而，对于 $\log \Box$ 的含义目前没有统一结论，这部分计算并不是用曲率表达的。有效作用量中对数项的修正涉及一般形式为 $R^2 \frac{1}{\Box} R$ 的结构。这些结构在圈展开和导数展开中都与对数修正处于同阶。量纲分析可以证明，在施瓦西情形下，视界附近这些修正和对数修正的幅度相同。因此即便是在这里我们也能看到微扰论的局限性。

In a less formal context, we can also see in the calculations above that there is not a universal quantum metric. We can look at the calculations and see if there is a corrected metric which would describe these situations. One can easily see that there is no single metric around a massive body which would recover the behavior for both the non-relativistic and massless particles. This result is also implied indirectly in the discussion of test particles. The diagrams that go into the calculation of a change in the metric, which would include Fig. 1a, are not the final result, which would also include Fig. 1b.

在更不形式化的层面，我们也可以从上述计算中看出，不存在普适的量子度规。我们可以查看这些计算，看看是否存在一个修正后的度规能够描述这些情况。不难发现，大质量天体周围不存在单一的度规，可以同时重现非相对论粒子和无质量粒子的行为。这个结论也在检验粒子的讨论中被间接暗示。那些用于计算度规变化的图 (包含图 1a) 并不是最终结果，最终结果还需要包含图 1b。

Light Cones/Penrose Diagrams Appear as Uncontrolled Approximations

光锥/彭罗斯图是不受控近似

It is perhaps redundant to point out that the results described above call into question many of our standard tools. If massless particles follow different trajectories, which one is it that defines the light cone? If there is an intrinsic fuzziness to this concept, how does one draw a proper Penrose diagram? To be sure, these effects are small in the limits where the EFT is valid. Using the standard tools in that regime will be approximately valid, with calculable corrections. But the effects get larger and more important as one approaches the limits of the EFT, where many of the interesting quantum gravity questions are posed. These tools are not controlled approximations outside of the EFT region.

或许无需赘言，上述结果已经对我们的许多标准工具提出了质疑。如果无质量粒子沿不同轨迹运动，那么究竟是哪一条轨迹定义了光锥？如果这个概念本身就存在内在模糊性，我们又该如何绘制出规范的彭罗斯图？可以肯定的是，在有效场论 (EFT) 适用的范围内，这些效应都很微弱。在该区域使用标准工具是近似成立的，还能计算出修正项。但当我们趋近有效场论的适用边界时，这些效应会变得越来越大、越来越重要，而诸多有趣的量子引力问题都正是在这个边界区域提出的。在有效场论的适用范围之外，这些工具都不属于受控近似。

Limits of the Gravitational Effective Field Theory

引力有效场论的极限

The heart of effective field theory is the idea that only the degrees of freedom which are active at a given energy need to be included and we only need to know their interactions near that scale. Physics is an experimental science, and there are boundaries to what we know. Here are some comments on the limits of the effective field theory for gravity.

有效场论的核心思想是: 只需包含给定能量下活跃的自由度，我们仅需了解这些自由度在该能标附近的相互作用即可。物理学是一门实验科学，我们的认知存在边界。下文将讨论引力有效场论框架的局限性。

High Energy

高能

The most obvious limit to the effective field theory is at high energy or large curvature. At some energy, our knowledge of the right degrees of freedom or of their interactions fails. We then need a more complete theory. Or perhaps if the same ingredients remain valid, near the Planck scale, we would enter a strongly coupled regime where EFT techniques would be useless.

有效场论最明显的适用极限出现在高能或大曲率条件下。到达一定能量后，我们对正确自由度及其相互作用的认知就会失效，这时我们需要一个更完备的理论。哪怕原有理论框架依然成立，在接近普朗克尺度时我们也会进入强耦合区域，有效场论方法在那里将不再适用。

Many of the most interesting question in the study of quantum gravity are sensitive to the high-energy limit. In EFT jargon, this is referred to as the need for a UV completion. Many of the other contributions to this volume describe these theories.

量子引力研究中绝大多数最具价值的问题都对高能极限敏感。用有效场论的术语来说，这意味着我们需要一个紫外完备理论。本论文集的其他多篇撰稿都对这些理论进行了介绍。

The Extreme Infrared Limit of the Theory

理论的极端红外极限

Effective field theory is meant to be best in the infrared, and there is no indication that this is not correct. However, there are technical limitations on what we can do with present techniques, which become most obvious in the extreme infrared. These are perhaps more interesting than the high-energy limitations, as they may lead to new techniques and perhaps new insights.

有效场论本应在红外区域表现最佳，目前也没有迹象表明这一结论不成立。但现有技术在应用中存在技术局限，这种局限在极端红外区域最为明显。这些局限或许比高能局限更值得研究，因为它们可能催生新方法，带来新见解。

The local effective action which we start with is an expansion in the local curvature. In general, the non-local quantum effects are calculated by a perturbative expansion around a background metric, $g_{\mu\upsilon} = \bar{g}_{\mu\upsilon}+\kappa h_{\mu\upsilon}$. However, gravitational effects build up, and the metric can get large even when the local curvature is small. By the equivalence principle, we can always chose the background metric to be almost flat in a neighborhood of any point. This is most clearly seen using normal coordinates, which expand about a position in spacetime

我们所用的局域有效作用量是对局域曲率做展开得到的。一般来说，非局域量子效应通过围绕背景度规 $g_{\mu\upsilon} = \bar{g}_{\mu\upsilon} + \kappa h_{\mu\upsilon}$ 的微扰展开计算。但引力效应会不断累积，即使局域曲率很小，度规也可以变得很大。根据等效原理，我们总可以选择背景度规，使其在任意点的邻域内近似平直。这一点用正规坐标最容易体现，正规坐标是围绕时空某一位置做展开得到的

$$g_{\mu\nu}(x') = \eta_{\mu\nu} + \frac{1}{3}R_{\mu\alpha\mu\beta}(x)\,y^{\alpha}y^{\beta} + \dots\,,\ y = (x' - x)\,. \tag{25}$$

However, if the distance away is far enough, the metric in this expansion will become large unless higher-order terms in the expansion, of order R^2 and higher, become important. An extreme example of this is the standard Schwarzschild coordinates for a black hole. Coordinates which are smooth at infinity have a metric which blows up at the horizon. If we choose coordinates which are smooth near a point on the horizon, they will blow up somewhere else. This happens even though the curvature itself will be small outside and on the horizon for massive black holes.

但如果距离展开点足够远，除非展开中 R^2 阶及更高阶的高阶项发挥作用，否则该展开下的度规会变得无穷大。一个典型例子就是黑洞的标准史瓦西坐标: 在无穷远光滑的坐标，其度规会在视界处发散；如果选择在视界上某点附近光滑的坐标，度规又会在其他地方发散。哪怕是大质量黑洞，其视界外部和视界上的曲率本身都很小，这种情况依然会发生。

This problem also permeates classical perturbation theory. One can use post-Newtonian expansions to calculate gravitational radiation for the far-field part of the inspiral phase, but we need numerical techniques to capture the results near the horizon scale even when the curvature there is small.

这个问题也存在于经典微扰论中。人们可以用后牛顿展开计算旋进阶段远场部分的引力辐射，但哪怕视界尺度的曲率很小，我们仍需要数值方法来得到视界尺度附近的结果。

However, the problem is more severe in quantum perturbation theory. We have seen that the quantum gravity effects are non-local because the massless propagators can probe large distances. The non-locality implies that even if we are in a region of small metric deviation, the propagators can probe the larger metrics further away. Most of our field theoretic techniques are adaptions from flat space methods. For a specific example, consider the logarithmic non-locality such as we have discussed frequently above. In flat space, dimensional analysis tells us that for time-independent problems

但该问题在量子微扰论中更为严重。我们已经知道，量子引力效应是非局域的，因为质量为零的传播子可以探测大尺度距离。这种非局域性意味着，即使我们所在区域的度规偏离量很小，传播子也能探测到更远区域的大度量。我们现有的多数场论技术都改编自平直空间方法。举一个具体例子，考虑我们上文反复讨论过的对数非局域性。在平直空间中，量纲分析告诉我们，对于定态问题

$$\langle \mathbf{x} | \log \Box | \mathbf{y} \rangle = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})} \log\left(\mathbf{q}^2\right) \sim \frac{1}{(\mathbf{x}-\mathbf{y})^3} \tag{26}$$

The resulting $1/r^3$ dependence is seen in the non-relativistic potential Eq. 20, the bending angle Eq. 21, and the metric correction Eq. 24. However, this means that if you were to use this form in the analysis of the Schwarzschild solution, even if you were at a large distance from the center, you would be sensitive to the horizon where the metric blows up and sensitive even to the curvature singularity at the origin.

由此产生的 $1/r^3$ 依赖可以在非相对论势 (式 20)、偏转角 (式 21) 和度规修正 (式 24) 中看到。但这意味着，如果将这种形式用于分析史瓦西解，哪怕你位于离中心很远的位置，结果也会受到度规发散的视界的影响，甚至会受到原点处曲率奇点的影响。

Again, this seems to be a technical problem, which could potentially be solved by numerical relativity. Intuitively, we expect such effects to be small - we should not need to know about the black hole at the center of our galaxy in order to do weak field calculations on earth. But we presently do not have a universal estimator for how big such corrections are. And we do not have any proof that the quantum effects do not build up over long distances like the classical effects do (Yang-Mills theory also is a good perturbation theory at short distances but, for very different reasons, at large distance becomes non-perturbative. At the very largest distances, QCD and QED in the real world have neutral states so that the growth with distance is not important. Gravitational charges are all the same sign, so effects can build up.). So this becomes a limitation on the EFT techniques.

这看起来仍是一个技术问题，或许可以通过数值相对论解决。直觉上，我们预期这类效应很小——在地球做弱场计算并不需要知道银河系中心的黑洞信息。但目前我们还没有通用方法估计这类修正的大小，也没有证据证明量子效应不会像经典效应一样在长距离上累积 (杨-米尔斯理论在短距离也是很好的微扰理论，但出于完全不同的原因，它在大距离会变成非微扰理论。在极大距离上，现实世界中的 QCD 和 QED 都对应中性态，因此效应随距离增长并不重要。而引力荷的符号全都相同，因此效应可以不断累积)。因此这成为了有效场论方法的一项局限。

Perhaps the issue could be addressed by combining patches using different coordinates and matching on the boundary. In each patch, we can use the equivalence principle to make the coordinates nearly flat. Then matching at the boundaries would convey the information from one patch to another. However, this program has not yet been carried out.

或许这个问题可以通过拼接不同坐标的 patches 并在边界匹配来解决。在每个 patch 中，我们都可以利用等效原理让坐标近似平直，再通过边界匹配传递不同 patch 之间的信息。但这套方案目前还尚未实施。

What Are the Right Quantum Questions?

什么是正确的量子问题?

Quantum field theory in curved spacetime is a challenging subject even for nongravitational particles and interactions. It is not even clear how to rigorously define a particle in general curved spacetimes. However, in lightly curved worlds, we can approximately use Minkowski field theory techniques. We do this all the time since we live in a lightly curved spacetime. In curved spacetime, the gravitational effective field theory shares these challenges and adds an extra one because the metric is now a dynamical variable.

弯曲时空的量子场论即使对非引力粒子和相互作用而言都是一个极具挑战的课题。在一般弯曲时空中，甚至连如何严格定义粒子都不清晰。不过，在轻度弯曲的时空背景下，我们可以近似使用闵可夫斯基场论技术。由于我们本身就生活在一个轻度弯曲的时空中，我们一直都在这么做。在弯曲时空里，引力有效场论不仅存在这些挑战，还多了一个额外难题: 度规本身现在是动力学变量了。

The perturbative solution is to expand around a background metric and treat the fluctuating field quantum mechanically. We have seen that this can yield leading predictions at low energy and curvature. But the effective theory has also demonstrated that some of our usual techniques have some limitations. Some of these can be traced to the perturbative expansion, as we have seen the difficulty of identifying the backreaction on the background metric itself.

微扰解法是围绕背景度规展开，然后对涨落场进行量子力学处理。我们已经知道，这可以在低能量和低曲率条件下给出领头阶预言。但该有效理论也表明，我们常用的部分技术存在一定局限性。正如我们所见，其中一些局限源于微扰展开本身——我们很难确定引力对背景度规本身的反作用。

The larger challenge is to identify valid quantum questions for gravity which are also able to be answered with techniques which we know how to apply. This may require the adoption of non-perturbative methods, at least in cases where the metric field becomes strong. Even in non-perturbative computational methods such as causal dynamical triangulations, it is also a challenge to identify sensible objects to be calculated [73]. The limits of perturbation theory itself may prove to be one of the limits of the effective field theory. In any case, there still are interesting field theory developments possible even within the effective field theory.

更大的挑战在于，如何找出对引力成立、同时还能用我们已知方法解答的有效量子问题。要做到这一点，至少在度规场变强的场景下，可能需要采用非微扰方法。即便是在因果动力学三角剖分这类非微扰计算方法中，确定值得计算的合理对象也是一项挑战 [73]。微扰论本身的局限，或许就会成为有效场论的局限之一。无论如何，哪怕是在有效场论的框架内，依然有诞生有趣场论新进展的空间。

Other Potential Limits

其他潜在极限

Effective field theory is a ”humble” theory in that it attempts to work within the framework of existing knowledge and to acknowledge its limits. In this regard, we should recognize that there are other limits on our understanding besides just the energy/distance variable.

有效场论是一种“谦逊”的理论，它始终在现有知识的框架内开展研究，并且明确承认自身存在局限。就此而言，我们应当认识到，除能量/距离变量外，我们的认知还存在其他局限。

In particular, quantum mechanics itself has only been tested within some limits. In reactions, there are generally only a few particles involved. Even in condensed matter physics, with enormous numbers of particles, the interactions are primarily two-body or sometimes three-body. We presently have no experimental need for modifications to quantum mechanics, as we do for other interactions outside their known frontiers. However, there could be a ”macroscopicity” frontier. Many issues in the transition from quantum physics to classical physics and decoherence remain poorly understood. Gravity could be the place where these effects become important, as gravity is not screened and very large masses are available. Recent work on stochastic effects and decoherence, reviewed by C. Burgess in this volume [74], provides some new techniques within quantum mechanics. But perhaps there could be real changes with quantum physics for objects that are macroscopic enough. This point of view has been put forward by Penrose [75] and by Stamp and collaborators [76].

具体来说，量子力学本身目前仅在部分范围内得到了检验。现有反应实验中，参与反应的粒子数通常很少。即便在粒子数量极多的凝聚态物理中，相互作用也主要是两体相互作用，有时最多是三体相互作用。目前和其他相互作用在其已知前沿外的情况不同，我们还没有在实验上发现修改量子力学的需求。但量子力学确实可能存在“宏观性”前沿。从量子物理到经典物理的转变以及退相干领域，许多问题至今仍未得到很好的理解。引力可能是这类效应显现的场所——引力无法被屏蔽，且可以存在质量极大的物体。本卷中 C. Burgess 综述的随机效应与退相干最新研究 [74]，在量子力学框架内提出了一些新方法。但对于足够宏观的物体，量子物理或许真的会发生改变。Penrose[75] 以及 Stamp 与其合作者 [76] 已经提出了这一观点。

Conclusion

结论

Claims that general relativity clashes with quantum mechanics are wrong or at least misleadingly simplistic. The covariant quantization of general relativity is by now an old topic [5,77-79]. Modern quantum field theory techniques - effective field theory - help us to extract physical quantum predictions. These techniques are completely normal quantum field theory methods which are applied routinely in other settings. So the quantum field theory of quantum general relativity is quite normal.

认为广义相对论与量子力学存在根本冲突的说法是错误的，或者至少是误导性的过度简化。广义相对论的协变量子化时至今日已是一个经典课题 [5,77-79]。现代量子场论技术——有效场论——可以帮助我们提取物理量子预言。这些技术都是完全标准的量子场论方法，已在其他领域常规应用。因此，量子广义相对论的量子场论框架是相当成熟自洽的。

Indeed, general relativity is perhaps the best example of the effective field theory paradigm. It appears to be valid over many orders of magnitude in distance or energy, and it overlaps with quantum physics in regions where the latter is relevant. We have tested the low-energy degrees of freedom, and their interactions follow from a simple action. Plus, since we do not know the ultimate UV completion for quantum gravity, the effective field theory is all that we can be sure about.

事实上，广义相对论或许是有效场论范式最完美的范例。它在距离和能量的多个数量级上都成立，并且在量子物理相关的区域与量子物理相容。我们已经对低能自由度进行了检验，其相互作用完全符合一个简单的作用量。此外，由于我们尚不清楚量子引力的最终紫外完备形式，有效场论是我们目前唯一可以确信的理论框架。

There is no reason to be misleadingly simplistic about quantum gravity. We have made strides in quantizing, renormalizing, and applying quantum general relativity. The effective field theory results are interesting in themselves. And we can readily motivate further quantum gravity work without being misleading. What we should be saying is that quantum general relativity points unmistakably to our lack of understanding of the full theory.

我们没有理由对量子引力做出误导性的过度简化。我们已经在量子化、重整化和应用量子广义相对论方面取得了长足进展。有效场论的结果本身就很有意义。我们完全不需要误导性表述也能充分推动量子引力领域的进一步研究。我们应当明确: 量子广义相对论清晰地表明，我们尚未理解完整的量子引力理论。

Quantum gravity is not optional. While we likely will not know the experimental outcome in our lifetimes regarding the various options, there is still much to learn about the consistency and structure of the theories. Perhaps the exploration will change our understanding of the world. But we do know that all of the theories of quantum gravity need to reduce to the effective field theory of general relativity in the appropriate limit. This effective field theory provides a foundation for our exploration of quantum gravity.

量子引力的研究不是可选项。尽管我们有生之年大概率无法看到不同理论方向的实验验证结果，但我们仍然可以探索这些理论的自洽性与理论结构。或许这一探索会彻底改变我们对世界的认知。但我们可以确定一点: 所有量子引力理论都必须在适当的极限下退化为广义相对论的有效场论。这个有效场论为我们探索量子引力提供了坚实基础。

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